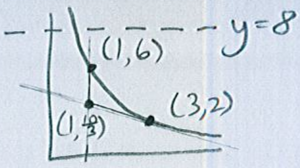
Consider the region defined by $y \le \dfrac{6}{x}$, $y \ge 4 - \frac{2}{3}x$ and $x \ge 1$.



SCORE: _____ / 14 PTS

$x = \frac{6}{y}$ $\quad x = \frac{3}{2}(4-y)$

[a] If the region is revolved around the line $y = 8$, write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid

$\frac{6}{x} = 4 - \frac{2}{3}x$

$18 = 12x - 2x^2$

$2x^2 - 12x + 18 = 0$

$2(x-3)^2 = 0$

$x = 3$

[i] using the disk or washer method **(NOTE: You do NOT need to simplify your integrand.)**

$$\pi \int_1^3 \left(\left(8 - \left(4 - \tfrac{2}{3}x\right)\right)^2 - \left(8 - \tfrac{6}{x}\right)^2 \right) dx$$

$\frac{1}{2}$

EXPLANATION: 1 FOR R

1 FOR r

1 FOR $R^2 - r^2$

☆

1 FOR EACH ITEM UNLESS OTHERWISE NOTED

[ii] using the shell method **(NOTE: You do NOT need to simplify your integrand.)**

$$2\pi \left(\int_2^{\frac{10}{3}} (8-y)\left(\tfrac{6}{y} - \tfrac{3}{2}(4-y)\right) dy + \int_{\frac{10}{3}}^{6} (8-y)\left(\tfrac{6}{y} - 1\right) dy \right)$$

$\frac{1}{2}$ $\quad \frac{1}{2}$

[b] Suppose the region is the base of a solid. Cross sections perpendicular to the $x-$ axis are isosceles right triangles with their hypoteneuse in the base. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

$\frac{1}{2}$

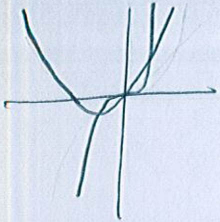
$$\frac{1}{4} \int_1^3 \left(\tfrac{6}{x} - \left(4 - \tfrac{2}{3}x\right) \right)^2 dx$$

$A = \frac{1}{4}s^2$

Find the area bounded by the curves $y = 4x^2 + 8x$ and $y = 4x^3$.

SCORE: _____ / 8 PTS

NOTE: Your final answer must be a number, not an integral nor sum of integrals.



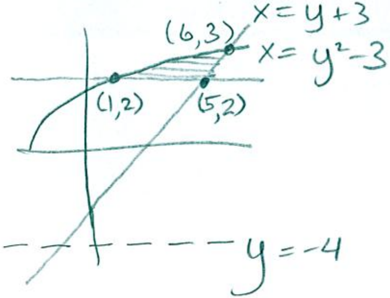
$$4x^2 + 8x = 4x^3$$
$$0 = 4x^3 - 4x^2 - 8x$$
$$0 = 4x(x-2)(x+1)$$
$$x = -1, 0, 2$$

1 EACH

$$\int_{-1}^{0} (4x^3 - (4x^2 + 8x))\,dx + \int_{0}^{2} (4x^2 + 8x - 4x^3)\,dx$$
$$= \left(x^4 - \tfrac{4}{3}x^3 - 4x^2\right)\Big|_{-1}^{0} + \left(-x^4 + \tfrac{4}{3}x^3 + 4x^2\right)\Big|_{0}^{2}$$
$$= 0 - \left(1 + \tfrac{4}{3} - 4\right) + \left(-16 + \tfrac{32}{3} + 16\right)$$
$$= \frac{37}{3}$$

The region bounded by $y=\sqrt{x+3}$, $y=x-3$ and $y=2$ is revolved around the line $y=-4$. **SCORE: _____ / 8 PTS**

Write, **<u>BUT DO NOT EVALUATE</u>**, an integral (or sum of integrals) for the volume of the solid **<u>using as few integrals as possible</u>**.



$y+3=y^2-3$

$\underbrace{0=y^2-y-6}$ 1

$0=(y-3)(y+2)$

$y=3, \cancel{-2}$

$$\underbrace{2\pi}_{2}\overset{1}{\int_2^3} \underbrace{(y+4)}_{2}\underbrace{(y+3-(y^2-3))}_{2}\,dy$$